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6.3 Antidifferentiation by Parts

Formula $\int udv = uv - \int vdu$

The idea is to make the solving the integral easier. If it looks like its getting messier – try something else or change the substitution. PRACTICE is the key here. This helps you to recognize different types of substitution.

Ex. 1 Evaluate $\int x\cos(5x-1)dx$

$\int udv = uv - \int vdu$

Let $u = x$ $\quad v = \frac{1}{5}\sin(5x-1)$

$du = dx$ $\quad dv = \cos(5x-1)dx$

$$\int x\cos(5x-1)dx = \frac{1}{5}x\sin(5x-1) - \frac{1}{5}\int \sin(5x-1)dx$$

$$= \frac{1}{5}x\sin(5x-1) + \frac{1}{25}\cos(5x-1) + C$$

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Ex. 2 Evaluate $\int \underset{u}{x^2} \underset{dv}{\cos 2x\,dx}$

Let $u = x^2$ $\quad v = \frac{1}{2}\sin 2x$

$du = 2x\,dx$ $\quad dv = \cos 2x\,dx$

$$\int x^2 \cos 2x\,dx = \frac{1}{2}x^2 \sin 2x - \int \frac{1}{2}\sin 2x \cdot 2x\,dx$$

$$= \frac{1}{2}x^2 \sin 2x - \int x \sin 2x\,dx$$

let $u = x$ $\quad v = -\frac{1}{2}\cos 2x$

$du = dx$ $\quad dv = \sin 2x\,dx$

$$= \frac{1}{2}x^2 \sin 2x - \left\{ -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x\,dx \right\}$$

$$= \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + C$$

$$\frac{1}{4}\left(2x^2 \sin 2x + 2x\cos 2x - \sin 2x\right) + C$$

TRY Evaluate $\int x \tan^{-1} x\,dx$

Let $u = \tan^{-1} x$ $\quad v = \frac{1}{2}x^2$

$du = \frac{dx}{1+x^2}$ $\quad dv = x\,dx$

$$\int x \tan^{-1} x\,dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

$$\begin{array}{r|l} & 1 \\ x^2+1 & x^2 \\ & x^2+1 \\ \hline & -1 \end{array}$$

$$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\left[\int dx - \int \frac{1}{1+x^2}\,dx\right]$$

$$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\left[x - \tan^{-1} x\right] + C$$

$$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + C$$

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Ex. 3 Solve the DE $\dfrac{dy}{dx} = x \ln x$ with IC's y=-1 when x=1.

Let $u = \ln x \quad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx \quad dv = x\,dx$

$$\int x \ln x\, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int \frac{x^2}{x}dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\, dx$$

$$y = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$$-1 = \frac{1}{2}\underbrace{\ln 1}_{0} - 1/4 + C$$

$$-\frac{3}{4} = C$$

$$\underline{y = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 - 3/4}$$

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Solving for an Unknown intergal - I really like this one :)

Ex. 4 Solve $\int e^x \cos x\,dx$

Let $u = e^x$ $v = \sin x$
$du = e^x dx$ $dv = \cos x\,dx$

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$

let $u = e^x$ $v = -\cos x$
$du = e^x dx$ $dv = \sin x\,dx$

$$\int e^x \cos x\,dx = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x\,dx\right]$$

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$+\int$ $+\int$

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x\,dx = \frac{e^x}{2}(\sin x + \cos x) + C$$

TRY

Solve $\int e^{3x} \sin 2x\,dx$

$u = e^{3x}$ $v = -\frac{1}{2}\cos 2x$
$du = 3e^{3x} dx$ $dv = \sin 2x\,dx$

$$\int e^{3x} \sin 2x\,dx = -\frac{1}{2} e^{3x} \cos 2x + \frac{3}{2}\int e^{3x} \cos 2x\,dx$$

$u = e^{3x}$ $v = \frac{1}{2}\sin 2x$
$du = 3e^{3x} dx$ $dv = \cos 2x\,dx$

$$\int e^{3x} \sin 2x\,dx = -\frac{1}{2} e^{3x} \cos 2x + \frac{3}{2}\left[\frac{1}{2} e^{3x} \sin 2x - \frac{3}{2}\int e^{3x} \sin 2x\,dx\right]$$

$$\int e^{3x} \sin 2x\,dx = -\frac{1}{2} e^{3x} \cos 2x + \frac{3}{4} e^{3x} \sin 2x - \frac{9}{4}\int e^{3x} \sin 2x\,dx$$

$$\frac{13}{4}\int e^{3x} \sin 2x\,dx = -\frac{1}{2} e^{3x} \cos 2x + \frac{3}{4} e^{3x} \sin 2x$$

$$\int e^{3x} \sin 2x\,dx = \frac{4}{13}\left[-\frac{1}{2} e^{3x} \cos 2x + \frac{3}{4} e^{3x} \sin 2x\right] + C$$

$$= \frac{e^{3x}}{13}\left[3\sin 2x - 2\cos 2x\right] + C$$

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Tabular Integration

We can use this method when the integral is in the form $\int f(x)g(x)dx$ and repeated differentiation would result in $f(x)$ becoming zero.

Ex. 5 Evaluate $\int x^2\cos 4x\,dx$

$\frac{d}{dx}$		$\int dx$
x^2	+	$\cos 4x$
$2x$	−	$\frac{1}{4}\sin 4x$
2	+	$-\frac{1}{16}\cos 4x$
0		$-\frac{1}{64}\sin 4x$

$\frac{x^2}{4}\sin 4x + \frac{2x}{16}\cos 4x - \frac{2}{64}\sin 4x + C$

$\frac{x^2}{4}\sin 4x + \frac{x}{8}\cos 4x - \frac{1}{32}\sin 4x + C$

TRY Evaluate $\int x^3 e^{-2x}dx$

$\frac{d}{dx}$		$\int dx$
x^3	+	e^{-2x}
$3x^2$	−	$-\frac{1}{2}e^{-2x}$
$6x$	+	$\frac{1}{4}e^{-2x}$
6	−	$-\frac{1}{8}e^{-2x}$
0		$\frac{1}{16}e^{-2x}$

$-\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - \frac{6}{8}xe^{-2x} - \frac{6}{16}e^{-2x} + C$

$-\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - 3/4\,xe^{-2x} - \frac{3}{8}e^{-2x} + C$

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Ex. 6 Find $\int \ln x dx$

$u = \ln x \quad v = x$

$du = \frac{dx}{x} \quad dv = dx$

$x\ln x - \int \frac{x}{x} dx$

$x \ln x - x + C$

Ex. 7 Evaluate $\int_0^1 \sin^{-1} x dx$

Let

$u = \sin^{-1} x \quad v = x$

$du = \frac{dx}{\sqrt{1-x^2}} \quad dv = dx$

$x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} dx$

$u = 1 - x^2$

$du = -2x dx$

$-\frac{1}{2} du = x dx$

$x\sin^{-1}x + \frac{1}{2}\int \frac{du}{\sqrt{u}}$

$x\sin^{-1}x + \sqrt{u} + C$

$x\sin^{-1}x + \sqrt{1-x^2} + C$

$x\sin^{-1}x + \sqrt{1-x^2} \Big]_0^1$

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